An Iterative Finite Element-Integral Technique for Predicting Sound Radiation from Turbofan Inlets in Steady Flight

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A new iterative solution technique for predicting the sound field radiated from a turbofan inlet in steady flight is presented. The sound field is divided into two regions: the sound field within and near the inlet which is computed using the finite element method, and the radiation field beyond the inlet which is calculated using an integral solution technique. A continuous solution is obtained by matching the finite element and integral solutions at the interface between the two regions. The applicability of the iterative technique is demonstrated by comparison of experimental and theoretical results for several different inlet configurations with and without flow. These examples show that good agreement between experiment and theory is obtained within five iterations.

Introduction

GROWING concern in recent years over the rise in noise pollution—especially that caused by turbofan engines—has created considerable interest in developing methods for predicting the sound radiated by these engines. The two major sources of turbofan engine noise are the externally generated noise produced by the jet exhaust and the internally generated noise due to rotating turbomachinery, turbofan blades, and combustion processes. The noise due to the jet exhaust has become less intense due to the use of high-bypass-ratio turbofan engines whose exhaust jet velocity is lower than that of earlier low-bypass-ratio engines. This, in turn, has made the noise produced by the turbofan more apparent. In an attempt to reduce the turbofan noise, much effort has been expended on the development of acoustic liners for these engine inlets. Unfortunately, due to the lack of appropriate analytical methods, most of the development work for these liners has been performed using costly cut-and-try methods. For this reason, analytical methods are needed to equip the designer with the tools necessary for the job of designing quieter turbofan inlets.

As the first step in the development of a theoretical approach for reducing the amount of noise that will reach an observer, whether it be a passenger in the aircraft or an observer on the ground, an analytical procedure for predicting the acoustic field of the engine must be developed. Such an analysis should account for the effects produced by changing the inlet length, inlet geometry and multidimensional steady flow, and inlet liner properties. To date, no such analytical method for predicting the entire sound field exists. This paper describes a general approach for calculating the acoustic field of a turbofan inlet in steady flight. The approach was developed from a procedure described by the authors in an earlier paper,[1] which considered static inlets. As in this earlier paper, the approach is not restricted to any particular computational schemes. Thus, many of the analytical approaches presently used in duct acoustics can be extended to provide a realistic description of the entire sound field.

Consider the inlet shown in Fig. 1. As the inlet is in forward flight, a steady flow of gas passes through and around the inlet. Away from the inlet the steady flow is uniform, while in the neighborhood of the inlet the steady flow is multidimensional with radial and axial gradients of flow properties. Noise produced within the engine propagates along the inlet duct through the multidimensional steady flow and radiates out to the surrounding environment.

These are two characteristics of this problem that greatly increase the difficulty in solving the appropriate acoustic equations: the presence of a multidimensional steady flow and the infinite domain of the problem. Mathematical techniques have been developed to treat each of these difficulties separately, however, neither approach can readily handle both difficulties. Even if it were possible to formulate a single mathematical approach capable of predicting the entire flowfield, the method would undoubtedly be very inefficient since the procedures required for incorporating multidimensional flow effects near the inlet would be redundant in the far field where the flow is uniform. Similarly, procedures developed for use far from the inlet would be inefficient within the inlet where most of the influence on the sound propagation is due to local constraints.

Thus, it is prudent to consider the development of a hybrid technique for solving the inlet sound radiation problem. This approach would divide the inlet acoustic field into interior and exterior regions. This division is governed by the requirement that all multidimensional flow effects be restricted to the "interior" region so that the infinite "exterior" region contains only a uniform steady flow. The solution for the entire acoustic field of an inlet in forward flight thus reduces to two separate, but dependent, acoustic problems: the determination of the sound propagation within a finite domain containing a multidimensional steady flow, and the determination of the sound radiation in an infinite domain in the presence of a uniform steady flow. Since these two solutions are not independent, the sound field must be continuous at the interface between the interior and exterior regions.

In a previous paper,[1] the authors described such a hybrid solution technique for inlets under static conditions. In the absence of a steady flow, the hybrid approach is not strictly necessary but was developed as a prelude to this paper. In the static case, the separation into interior and exterior regions was accomplished at the inlet entrance plane. Solutions for the acoustic field within the inlet were obtained us-

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ing a finite element method, while an integral technique was used to obtain solutions in the exterior region. Solutions for each of the regions require specification of a boundary condition on the interface between the two regions. Since this interface is an artificial boundary, no boundary condition based on physical conditions exists at this interface. Therefore, an iteration procedure was developed that improves the "boundary conditions" at this interface at each subsequent iteration until the acoustic solutions match at the interface and a continuous solution for the entire acoustic field is obtained.

Separation of the sound field into two acoustic regions is a standard procedure in turbofan acoustics. Typically this separation takes place at the inlet entrance. However, most analyses of the acoustic regions to date have considered the acoustic properties of these two regions separately without attempting to match the solutions at an interface and obtain a continuous description of the acoustic field. One classical example of this approach is the work of Tyler and Sofrin.[2] In their analysis, the inlet was taken to be a straight duct with the separation of the acoustic field coinciding with the end of the duct (i.e., the inlet entrance of Fig. 1). For the interior problem, it was assumed that there was no reflection of sound from the end of the inlet, and the method of separation of variables was used to obtain exact solutions. This "exact" solution within the duct predicts the acoustic pressure and velocity at the inlet entrance. This pressure at the entrance is then used to construct a simple radiation model for calculating the sound field in the exterior region. This radiation model is based on the far-field approximation of a piston in an infinite baffle. However, the impedance of the piston used in the exterior region differs from the no-reflection impedance assumed for the interior solutions. This indicates a discontinuity in the acoustic velocity at the entrance of the duct. No attempt is made by Tyler and Sofrin to remove this discontinuity.

Under certain conditions, Tyler and Sofrin's assumption of no reflection at the exit provides a reasonable approximation, which is used to derive the inlet entrance plane boundary condition. Unfortunately, many computational approaches (finite element, finite difference) do not separate the acoustic waves into individual modes. In these cases, the correct boundary condition for no reflection of plane waves will produce a reflection of higher order transverse or radial modes.

One alternative to the no-reflection impedance is the use of experimentally determined values of the exit impedance. In their finite element approach, Tag and Lumsdaine[3] used an empirical formula based on previous experimental results[4] to include Mach number effects at the inlet entrance. This approach provides the correct reflection for a single mode, however, as in the no-reflection case, unless a mode-matching approach is used, any additional modes present in the duct will not be reflected correctly.

Baumeister[5] has argued that the two regions are usually not strongly coupled, and the discontinuities can be removed by repeated calculation of the interior and exterior acoustic regions until the two solutions match at the interface. Results presented in Ref. 1 show that this is indeed the case; even the simplest iteration scheme converges rapidly.

The purpose of this paper is to extend the procedure for computing the complete acoustic field of a static turbofan inlet described in Ref. 1 to the case of inlets in forward flight. The previous approach utilized the following four steps: 1) separation of the acoustic region into interior and exterior regions with the interface at the inlet entrance; 2) solution for the interior acoustic field using a simple finite element method[6] with linear elements; 3) solution of the exterior acoustic field using an integral technique[7]; and 4) an iteration scheme using successive substitution to match the two acoustic solutions at the interface and obtain a continuous acoustic solution. In the presence of a steady flow, the location of the interface (see Fig. 1) is dictated by the requirement that all flow gradients occur within the interior region. It should be noted that the farther the interface is from the inlet, the better the assumption of no reflection becomes. An improved finite element method using Hermitian elements is used to solve the acoustic equations in the interior region. In the exterior region, the acoustic equations with a uniform steady flow are transformed to the Helmholtz equation so that the integral technique[7] could be applied in this case. Finally, the iteration pocedure used in Ref. 1 is shown to lead to a converged solution in relatively few iterations.

Problem Formulation

Formulation of the problem involves specification of the appropriate acoustic equations and boundary conditions, a description of the inlet geometry and steady flow conditions, and an outline of the mathematical approaches.

Acoustic Equations

Consider the inlet shown in Fig. 1. The flow of gas is assumed to be inviscid, non-heat-conducting, and irrotational. Although these assumptions exclude consideration of the effects of boundary layers in the mean flow on the acoustic properties, it assures the existence of a velocity potential with a corresponding decrease in the number of dependent variables. Body forces are neglected. To derive the needed nondimensional conservation equations, velocities, lengths, and time are respectively normalized with respect to the sound speed at stagnation conditions c_0^*, duct radius d_r^*, and d_r^*/c_0^*. The density ρ^* and pressure p^* are normalized with the stagnation density ρ_0^* and $\rho_0^* c_0^{*2}$, respectively. The velocity potential φ^* is normalized with respect to $c_0^* d_r^*$, and the frequency with respect to c_0^*/d_r^*. Under these conditions, it can be shown that the behavior of the flow in the duct is described by the following nonlinear partial differential equation for the flow potential φ:

$$\frac{\partial^2 \varphi}{dt^2} + \frac{\partial}{\partial t}(\nabla\varphi\cdot\nabla\varphi) + \frac{1}{2}\nabla\varphi\cdot\nabla(\nabla\varphi\cdot\nabla\varphi) = c^2\nabla^2\varphi \tag{1}$$

where

$$c^2 = 1-(\gamma-1)\left[\frac{\partial\varphi}{\partial t} + \frac{1}{2}\nabla\varphi\cdot\nabla\varphi\right]$$

and γ is the ratio of specific heats. Rewriting Eq. (1) in a cylindrical coordinate system (i.e., r, θ, z) with the z axis coinciding with the turbofan inlet axis yields

$$c^2\left\{\varphi_{rr} + \frac{\varphi_r}{r} + \frac{\varphi_{\theta\theta}}{r^2} + \varphi_{zz}\right\} - \varphi_{tt} = 2\varphi_r\varphi_{rt} + \frac{2\varphi_\theta\varphi_{\theta t}}{r^2}$$
$$+ 2\varphi_z\varphi_{zt} + \varphi_r^2\varphi_{rr} + \frac{\varphi_\theta^2\varphi_{\theta\theta}}{r^4} + \varphi_z^2\varphi_{zz} + \frac{2\varphi_r\varphi_\theta\varphi_{r\theta}}{r^2}$$
$$+ 2\varphi_r\varphi_z\varphi_{rz} + \frac{2\varphi_z\varphi_\theta\varphi_{z\theta}}{r^2} - \frac{\varphi_r\varphi_\theta^2}{r^3} \tag{2}$$

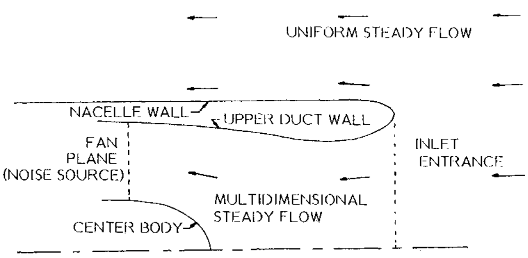

Fig. 1 Geometry of an inlet in flight.

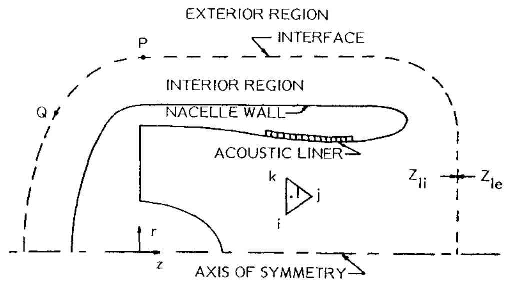

Fig. 2 Geometry of an idealized inlet.

where the subscripts indicate partial differentiation with respect to the subscripted variables.

To obtain the acoustic equations, the flow potential is rewritten as the sum of a steady axisymmetric mean flow potential $\bar{\varphi}(r,z)$ and an acoustic potential $\varphi'(r,\theta,z,t)$:

$$\varphi(r,\theta,z,t) = \bar{\varphi}(r,z) + \varphi'(r,\theta,z,t) \tag{3}$$

Because of the rotational nature of the fan and compressor,[2] they tend to generate sound characterized by acoustic modes rotating in a single direction. Thus only half of the possible spinning eigenfunctions of the duct will be considered by assuming the acoustic potential to be of the following form:

$$\varphi'(r,\theta,z,t) = \varphi(r,z)e^{-i(\omega t - m\theta)} \tag{4}$$

where $\varphi(r,z)$ is a complex quantity; i.e.,

$$\varphi = \tilde{\varphi} + i\hat{\varphi} \tag{5}$$

Substitution of Eq. (3) into Eq. (2) produces a lengthy equation consisting of three types of items: 1) terms containing only products of steady or mean flow quantities, 2) products of mean flow quantities and acoustic quantities, and 3) products of acoustic quantities. Since the mean flow is assumed to be undisturbed by the presence of the acoustic perturbations, the terms containing only products of mean flow quantities describe the behavior of the mean flow and their sum is identically zero. Furthermore, products of acoustic quantities are neglected in comparison to products of mean and acoustic quantities. The remaining equation, consisting of the products of the mean and acoustic quantities, is referred to as the linearized acoustic equation. Substituting Eqs. (4) and (5) into the linearized acoustic equation leads to derivation of the following linear partial differential equation:

$$F(\varphi) = A_1\varphi_{rr} + A_2\varphi_{zz} + A_3\varphi_{rz} + A_4\varphi_r + A_5\varphi_z + A_6\varphi = 0 \tag{6}$$

where

$$A_1 = \bar{c}^2 - \bar{\varphi}_r^2, \quad A_2 = \bar{c}^2 - \bar{\varphi}_z^2, \quad A_3 = -2\bar{\varphi}_r\bar{\varphi}_z$$

$$A_4 = -(\gamma+1)\bar{\varphi}_{rr}\bar{\varphi}_r - 2\bar{\varphi}_{rz}\bar{\varphi}_z + (\bar{c}^2/r)$$

$$- (\gamma-1)(\bar{\varphi}_r^2/r) - (\gamma-1)\bar{\varphi}_r\bar{\varphi}_{zz} - i[2\omega\bar{\varphi}_r]$$

$$A_5 = -(\gamma+1)\bar{\varphi}_{zz}\bar{\varphi}_z - 2\bar{\varphi}_{rz}\bar{\varphi}_r - (\gamma-1)\bar{\varphi}_{rr}\bar{\varphi}_z$$

$$- (\gamma-1)(\bar{\varphi}_z\bar{\varphi}_r/r) - i[2\omega\bar{\varphi}_z]$$

$$A_6 = \omega^2 - (m^2\bar{c}^2/r^2) - \{\omega(\gamma-1)[\bar{\varphi}_{rr} + (\bar{\varphi}_r/r) + \bar{\varphi}_{zz}]\}$$

Since the steady flow is assumed to be axisymmetric, the equation expressing conservation of acoustic momentum in the θ direction can be integrated to give the following relationship between the pressure perturbation p and the velocity potential φ:

$$p = -\bar{\rho}(i\omega\varphi + \bar{\varphi}_z\varphi_z + \bar{\varphi}_r\varphi_r) \tag{7}$$

which is valid only when both the steady and acoustic fields are irrotational.

Boundary Conditions

To complete the problem formulation, the boundary conditions for the problem must be specified. As previously noted, sound generated at the fan plane propagates down the duct "against" the multidimensional steady flow and radiates out to the environment. To prescribe the boundary conditions, the region of interest is divided into a finite interior region and an infinite exterior region, as shown in Fig. 2. The rear portion of the inlet is replaced by an ellipsoidal termination since it has been previously established that the geometry of this region has very little effect on the sound radiated from the inlet.[7]

Interior Region

It has been noted previously that the interface should be chosen so that the steady flow is uniform in the exterior region. Computer storage limitations necessitate a compromise and the interface is located so that derivations from a uniform flow in the exterior region are less than 5% and are neglected.

As shown in Fig. 2, the boundaries of the interior region are: the axis of symmetry, the centerbody, the fan plane, the upper duct wall, the nacelle wall, and the interface. As noted in previous papers on finite elements,[8] the boundary conditions enter the computations through surface integrals. Thus, it is convenient to locate an orthogonal coordinate system (s,n) on the boundary of the inlet so that the unit vector n points into the region and s traverses the boundary in a counterclockwise direction. As discussed in Ref. 6, the finite element formulation requires specification of the acoustic velocity component normal to each of the boundaries (i.e., φ_n). Along the axis of symmetry and the acoustically hard walls (centerbody, part of the upper duct wall, and the nacelle wall) the boundary condition is

$$\varphi_n = 0 \tag{8}$$

Along a portion of the upper duct wall (see Fig. 2) a point-reacting liner may be prescribed. Myers[9] has shown that a proper formulation of the requirement of continuity of normal velocity yields

$$\frac{\partial\varphi}{\partial n} = -\frac{p}{Z_1} - \frac{1}{i\omega}\bar{\varphi}_s\frac{\partial(p/Z_1)}{\partial s} + \left(\frac{p}{i\omega Z_1}\right)\bar{\varphi}_{nn} \tag{9}$$

where p is given in Eq. (7). In this analysis, it is assumed that the last term in Eq. (9) is negligible since $\bar{\varphi}_{nn}$ is quite small and cannot be accurately calculated by the method currently used for computing the steady flow.

The fan plane is the interface between the inlet and the turbofan engine and is therefore, the location where the noise enters the duct. Thus, the noise source at the inlet exit is given by

$$\varphi_n = f(r) \tag{10}$$

For axially symmetric plane wave excitation, $f(r)$ equals a constant. The interface that encloses the inlet artificially separates the acoustic field into an interior and an exterior region. Since a physical boundary condition does not exist at this location, the normal acoustic velocity and acoustic pressure can be related by the following impedance condition:

$$Z_{Ii} = -p/\varphi_n \tag{11}$$

where the subscript i represents the "interior" value of the impedance. Since the value of Z_{Ii} is not known a priori, an iteration scheme is used to determine the correct value of Z_{Ii}.

Exterior Region

The boundaries of the exterior region consist of the axis of symmetry and the interface. On the axis of symmetry, Eq. (8) holds. Along the interface an axially symmetric acoustic potential is introduced

$$\varphi(s) = g(s) \tag{12}$$

As will be described later, values of $g(s)$ are obtained from the interior solution and they change with each iteration.

Finally, in the proper physical description of the problem, no sound would be emitted before the engine was started at time t_0, and thus no disturbances would exist beyond $R = (\bar{\varphi}_x + c_0)(t - t_0)$. In the stationary formulation, this restriction is stated by the Sommerfeld radiation conditions.[7] In the integral technique, these conditions are met by choosing the Green's functions that satisfy these conditions.

Iteration Scheme

With the form of the boundary conditions specified, it is possible to formulate a simple iterative scheme. Specification of a physical problem involves: 1) a description of the inlet geometry, 2) specification of the location and impedance values of acoustic liners (if present), and 3) the nature of the sound source [i.e., $f(r)$]. With these conditions specified, all boundary conditions for the interior region are known except for the value of the impedance at the interface Z_{Ii}. In the exterior region, all boundary conditions are known with the exception of the acoustic potential at the interface [i.e., $g(s)$]. A simple iterative procedure based on the method of successive substitution can be formulated as follows:

1) Employing the no-reflection impedance condition ($Z_{Ii} = 1.0$), use the finite element method to obtain a solution within the interior region. From this solution compute the acoustic potential at the interface.

2) Let the acoustic potential at the interface as computed in step 1 equal $g(s)$. Use the integral technique to compute the exterior field. From the exterior solution, compute the impedance at the entrance plane (i.e., Z_{Ie}), where the subscript e denotes the external impedance at the interface.

3) If Z_{Ie} is not equal to Z_{Ii}, set Z_{Ii} equal to Z_{Ie} and repeat the solution of the interior region using the finite element method and obtain a new acoustic potential, $g(s)$, at the interface.

4) Using the $g(s)$ calculated in step 3, repeat the solution of the exterior region using the integral technique and obtain a new interface impedance, Z_{Ie}.

5) Repeat steps 3 and 4 as necessary until the impedance used in step 3, Z_{Ii}, agrees with the impedance calculated in step 4, Z_{Ie}, within a specified tolerance.

Computational Methods

The finite element method and the integral technique used in this paper are described in detail elsewhere.[6,7]

However, several improvements have been made to the finite element computation procedure, and a transformation has been utilized so that the integral formulation of Meyer et al.[7] can be used. Details of these improvements as well as a brief description of each method are provided in this section. It should be noted that other computational schemes such as finite difference methods[11,12] or other integral approaches[13] could also be used in this iteration scheme. Mode-matching methods are not appropriate since the interior region now includes a sizable region outside the inlet duct.

Steady Flow

The procedure used for computing the steady flow through and around the turbofan inlet is described in Ref. 6. Since the steady flow equations are nonlinear, an approximate solution technique was used. The incompressible steady flow (linear) equations were solved using the standard integral technique developed by Hess and Smith.[14] Next, a semiempirical correction[15] was applied to the incompressible results to produce a "compressible" solution, which was used as an input to the linear acoustic analysis.

Finite Element Method

Solution of Eq. (6) by the finite element method (FEM) requires a reformulation of the differential equation as an integral equation. As discussed in Ref. 6, no variational principle corresponding to Eq. (6) has been established, and it has been necessary to use a Galerkin approach. Following Ref. 6, Galerkin's form of the method of weighted residuals can be written as

$$\sum_{e=1}^{E} \iint N_m^e F(\varphi) r \, \mathrm{d}r \, \mathrm{d}z = 0, \quad m = 1,2,\ldots,M \tag{13}$$

where N_m^e is the weighting function for mode m over element e, and E is the total number of modes for element e. The acoustic equation, denoted by the operator $F(\varphi)$, is given by Eq. (6).

The interior region (see Fig. 2) is subdivided into triangular elements. Previous papers by Sigman and Zinn[8] have used linear[6] and quadratic[10] elements. It was shown in Ref. 10 that linear elements cannot be used when steady flows and acoustic liners are present since the linear elements cannot satisfy Eq. (9). Although the quadratic elements can be used, the computations described in this paper used the powerful Hermitian elements.[16] This element matches values of the potential and the derivatives of the potential (velocities) at each vertex of the triangles. The interpolating function for the Hermitian element is written as follows:

$$\varphi(r,z) = N_1^e\varphi_i + N_2^e\varphi_{zi} + N_3^e\varphi_{ri} + N_4^e\varphi_j + N_5^e\varphi_{zj} + N_6^e\varphi_{rj} + N_7^e\varphi_k + N_8^e\varphi_{zk} + N_9^e\varphi_{rk} + N_{10}^e\varphi_l \tag{14}$$

where φ_i, φ_{zi}, and φ_{ri} represent values of the potential, axial velocity, and radial velocity, respectively, at the ith node. The indices i, j, and k represent the vertices of the triangle and the index l denotes the centroid. Values of the weighting functions N_1^e, N_2^e,... are given in Ref. 16.

When linear elements are used, Eq. (14) can be substituted into Eq. (13) and the integrations can be readily performed. The integrations become more tedious when quadratic elements are used, but they are still within reason.[17] When Hermitian elements are used, analytic integration of the element equations becomes formidable. In this case it is far more practical to evaluate the integrals numerically using a Gaussian quadrature scheme. Since the weighting functions are polynomials, the Gaussian quadrature is exact,[18] and the change of human error is substantially reduced by letting the computer perform the integration. Furthermore, once the quadrature scheme is coded, the finite element computer codes can be changed to linear or quadratic elements with a minimum of reprogramming. The remaining tasks of assembling and solving the matrix equation follow the general approach of finite element methods.[1,16] In this case, however, the acoustic velocities are contained in the solution vector and need not be calculated from the potential.[16]

Integral Technique

The term "integral technique" is used to denote a procedure that uses Green's theorem to transform the problem of solving a partial differential equation within a region to an equivalent problem of solving an integral equation over the boundary of the region. For acoustic problems, the integrals vanish at infinity and on the axis of symmetry leaving only the integral over the body surface, or, in this case, the "interface" (see Fig. 2). Integral techniques for the Helmholtz equation have been developed by Meyer et al.,[7] Hess,[13] and Bell et al.[19] Meyer's approach was used in this study since the computer codes were readily available and the codes' accuracy has been demonstrated repeatedly.[7,20]

Use of these computer codes required the transformation of the acoustic equation applicable to situations involving a uniform steady flow to the Helmholtz equation. Such a transformation has been developed by Bell[21] and Tester,[22] and it is described below.

If the steady flow is uniform and axial, Eq. (6) reduces to

$$\bar{F}(\varphi)=\varphi_{rr}+\frac{\varphi_r}{r}+[1-M^2]\varphi_{zz}+\left[k^2-\frac{m^2}{r^2}\right]\varphi + 2ikM\varphi_z=0 \tag{15}$$

where

$$k=\omega/\bar{c} \text{ and } M=\bar{\varphi}_z/\bar{c}$$

Using the transformations:

$$\tilde{r}=r, \quad \tilde{z}=z/\sqrt{1-M^2}, \quad \tilde{k}=k/\sqrt{1-M^2}, \quad \tilde{\varphi}=\varphi e^{-i\delta\tilde{z}} \tag{16}$$

where

$$\delta=-kM/\sqrt{1-M^2}$$

it can be readily shown that Eq. (15) reduces to the following Helmholtz equation:

$$\tilde{\varphi}_{\tilde{r}\tilde{r}}+\frac{\tilde{\varphi}_{\tilde{r}}}{\tilde{r}}+\tilde{\varphi}_{\tilde{z}\tilde{z}}+\left[\tilde{k}^2-\frac{m^2}{\tilde{r}^2}\right]\tilde{\varphi}=0 \tag{17}$$

The previously specified boundary condition

$$\varphi(s)=g(s)$$

on the interface, can be written in terms of $\tilde{\varphi}(\tilde{s})$, using Eq. (16). As shown by Meyer et al.,[7] Green's theorem can be used to rewrite Eq. (17) in the following integral form:

$$\int_0^1 \Phi(Q)\{K_1(P,Q)+K_2(P,Q)\}\mathrm{d}Sq - \Phi(P)\int_0^1\{F_1(P,Q)+F_2(P,Q)\}\mathrm{d}Sq - \int_0^1 \frac{\partial\Phi(Q)}{\partial n}\{I_1(P,Q)+I_2(P,Q)\}\mathrm{d}Sq = 2\pi\left\{\Phi(P)+\frac{i}{k}\frac{\partial\Phi(P)}{\partial n}\right\} \tag{18}$$

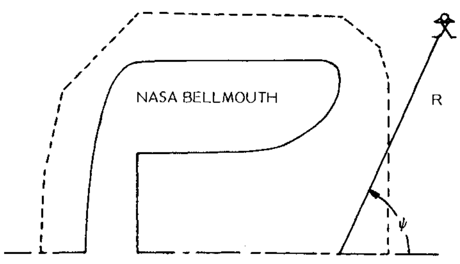

Fig. 3 Geometry of NASA bellmouth inlet and observer.

where

$$\Phi=\tilde{\varphi}e^{im\theta}$$

As shown in Fig. 2, P is a specific point on the body and Q is the integration point which traverses the body surface. The functions K_1, K_2, F_1, F_2, I_1, and I_2 are circumferential integrals which are functions of the body geometry and Green's function. For example,

$$K_1(P,Q)=2\int_0^\pi \frac{\partial G(P,Q)}{\partial n_Q}(e^{im\theta})\mathrm{d}\theta_Q \tag{19}$$

Green's function, $G(P,Q)$, is

$$G(P,Q)=\frac{\exp[ikR(P,Q)]}{R(P,Q)} \tag{20}$$

where

$$R(P,Q)=\sqrt{(z_p-z_Q)^2+(r_p-r_Q)^2} \tag{21}$$

Note that Eq. (20) satisfies the Sommerfeld conditions.

Equation (18) is reduced to a set of N linear algebraic equations by subdividing the boundary in Fig. 2 into N segments. The point P is fixed at the midpoint of a segment and the required integrations are performed over the boundary to produce an algebraic equation. By letting P be the midpoint of each of the N segments, N algebraic equations are obtained. The square matrix representing the N algebraic equations is inverted using standard matrix methods.

Results

The accuracy of the method is tested by comparison with experimental data obtained from tests conducted using a NASA bellmouth inlet with and without steady flow.[23]

For the calculations reported in this paper, the finite element computer codes subdivided the interior region into 54 Hermitian elements. These elements use 102 nodal points (triangle vertices and midpoints) which represent 198 degrees of freedom (i.e., unknowns). The computer codes for the integral technique subdivided the interface into 22 segments. In general, convergence was more rapid with the interface exterior to the inlet than in the previous study[1] where the interface was at the inlet entrance plane.

The NASA bellmouth inlet,[23] shown in Fig. 3, is a standard inlet used to evaluate the acoustic performance of various inlet shapes. Also shown in Fig. 3 is the coordinate system used to describe the location of an observer in the far field.

In order to compare the predictions of the hybrid finite element-integral technique solutions with experimental results, two series of tests were chosen from Ref. 23. The first series considers plane axisymmetric ($m=0$) and spinning mode ($m=2$) propagation in the bellmouth without steady flow. The second series of comparisons considers plane axisymmetric and spinning mode propagation with suction through the inlet. Although the inlet is static, the suction produces a multidimensional steady flow inside the inlet.

For the first series of comparisons, calculations were carried out for axisymmetric plane wave propagation at a frequency of $kr_0=1.39$, where r_0 is the dimensionless radius of the fan plane. As previously noted, there is no steady flow in the inlet. Figure 4 shows the variation of sound pressure level (SPL) as a function of the angle from the inlet centerline. The results of the experiments and the computer predictions are plotted as the SPL referenced to the peak value. This procedure results in the peak SPL being zero and the remainder of the sound field having a negative SPL. Several discrepancies should be noted in the experimental data of Ref. 23. First, the SPL is not a smooth curve, but exhibits a series of peaks and valleys, which Ville and

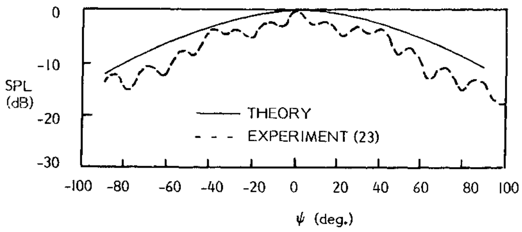

Fig. 4 Near-field sound pressure level of bellmouth inlet with no steady flow: $kr_0 = 1.39$; $m = 0$.

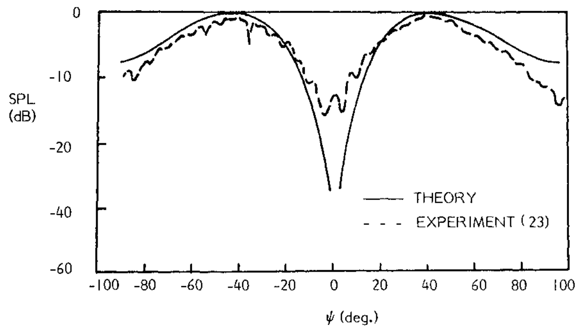

Fig. 5 Near-field sound pressure level of bellmouth inlet with no steady flow: $kr_0 = 3.37$; $m = 2$.

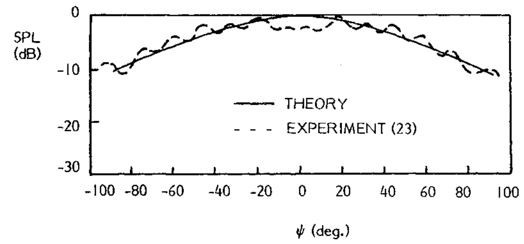

Fig. 6 Near-field sound pressure level of bellmouth inlet with inflow Mach number 0.4: $kr_0 = 1.39$; $m = 0$.

Silcox[23] attribute to resonance within the anechoic chamber. Note also that the plot of SPL is not symmetric as would be expected in this axisymmetric situation. These are not serious errors, but rather serve as a guide for judging the accuracy of the experimental data. Thus, it can be seen from Fig. 4 that the agreement between the theoretical results and experimental data is generally within the accuracy of the experiment. Figure 5 illustrates the variation of SPL for spinning wave ($m = 2$) propagation at a frequency $kr_0 = 3.37$. Once again, good qualitative agreement between the predictions of this study and the experimental data is demonstrated.

The second series of comparisons with the bellmouth inlet included the effects of multidimensional steady flows in the inlet. As noted, the experimental apparatus used a suction at the fan plane to induce a steady flow through the inlet. Corresponding calculations of the incompressible steady flow prescribed an inflow at the fan plane and zero freestream velocity.[14] Compressibility corrections were applied to produce a Mach number of 0.4 at the inlet fan plane. Figures 6 and 7 show comparisons of the SPL for axisymmetric plane wave propagation at $kr_0 = 1.39$ and spinning wave ($m = 2$) propagation at $kr_0 = 3.37$, respectively. The agreement is excellent for plane wave propagation, but two discrepancies appear in the spinning wave results. The falloff in the SPL of the spinning waves is greater than the computed values, and there appears to be a change in directivity of the maximum SPL of about 10 deg.

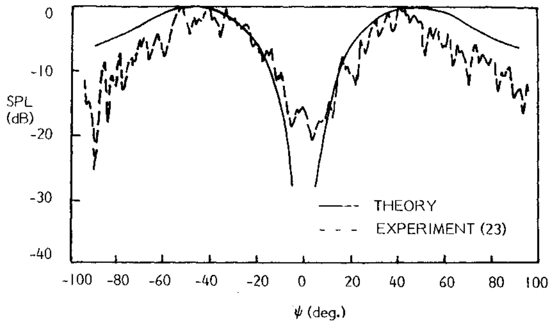

Fig. 7 Near-field sound pressure level of bellmouth inlet with inflow Mach number 0.4: $kr_0 = 3.37$; $m = 2$.

Since there is no reason to expect a noticeable change in directivity with the addition of a steady inflow, the computed results are subject to question. It appears that this discrepancy is due to a rather poor representation of the steady inflow for this particular inlet. Computational restrictions limited the number of segments that could be used to model an inlet surface in this study. Thick inlets, such as the NASA bellmouth inlet, required the use of larger and less accurate segments to model the inlet, thus producing a poor approximation to the steady flow over the exterior of the inlet. It is worth noting that these computer codes were also executed[24] on a larger computer using a more refined computational mesh (59 surface segments and 320 finite elements). With the finer mesh, the far-field SPL levels showed no change in directivity for the maximum SPL for the static and steady inflow cases. Thus, it appears that the change in directivity is a result of the coarse computational grid used in this paper.

Conclusions

The problem of sound propagation in finite length ducts and subsequent radiation to the external environment has been considered. The sound field is divided into a finite interior field in and around the inlet duct and an infinite exterior field. The interior acoustic field in the presence of a multidimensional steady flow is calculated using a finite element method with triangular Hermitian elements. A transformation is used to reduce the acoustic equations with uniform steady flow to the Helmholtz equation so that an integral technique can be used in the infinite exterior region. An iterative procedure is presented that forces the interior and exterior acoustic solutions to match at the interface, thus producing a continuous acoustic field.

The iterative finite element-integral technique was compared with experimental results for a NASA bellmouth inlet for axisymmetric plane and spinning wave propagation, with and without steady flow. In most cases, the theoretical results were in excellent agreement with measured results. Discrepancies in the directivity of the maximum SPL were eliminated using a refined computational mesh.

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